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Transport Property Correlations for H₂-He Gas Mixtures at Temperatures of 1000-25,000 K

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Two distinct correlations based on the results of a detailed H₂-He transport property analysis have been developed. The first correlation provides the transport properties of the individual species with mixture properties computed by existing semiempirical methods. The second correlation provides mixture transport properties for viscosity, Prandtl number, and thermal conductivity at equilibrium conditions. The validity of the two correlations is established by comparison with results of the detailed analysis. The influence of transport properties on predicting the no-injection convective and radiative heating rates for Jovian entry conditions is discussed.

Nomenclature

t, = total thermal conductivity, W/m-K

p = pressure, atm

 $Q_n = \text{transport property of mixture}$

 $Q_{p,i}$ = transport property of individual species

= temperature, K

Introduction

THE flowfields about blunt bodies entering the atmospheres of the outer planets are predicted by detailed computer codes based on both viscous 1,2 and inviscid 3 flow assumptions. In addition, a reliable approximate inviscid code⁴ has been developed and is suitable for parametric studies. The viscous shock layer (VSL) analyses 1,2 provide a direct means for computing the radiative and convective heat fluxes as well as any interactions between inviscid and viscous flow regions due to heat transfer, entropy-layer swallowing, and mass injection due to heat-shield ablation. The inviscid radiative analyses 3,4 must incorporate some separate convective heating code to account for viscous effects. Published results 5,6 have indicated that convective heating would be of importance only during the latter part of the heating pulse for Jupiter entry,5 but may be the dominant heating mode for a Saturn or Uranus entry.

An inherent requirement in the VSL codes and a necessary factor in most convective heating analyses is the definition of the transport properties. The influence of transport properties on convective heating calculations is well documented, e.g., a recent study indicated a 25-40% difference in convective heating rates for Jupiter entry conditions where the calculations were based on two sets of hydrogen-helium

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transport properties. Certainly, such discrepancies indicate the need of a detailed transport calculation for hydrogenhelium gas mixtures. Such a procedure utilizing the most current and reliable data was recently developed by Biolsi. ⁸ However, this detailed analysis would require large computer run times and storage, and thus it would not be advantageous to include the analysis in existing flowfield solutions.

The purpose of this paper is to present two distinct correlation procedures based on the results of the detailed transport property analysis for H₂-He gas mixtures.⁸ The first method is a correlation procedure developed to compute the transport properties of the individual species. Required mixture properties can be computed by existing semiempirical mixing techniques. The second method considers only the equilibrium mixture and provides correlations based on the analysis of Ref. 8 for viscosity, thermal conductivity, and Prandtl number. Correlations for both the frozen and total (frozen plus reactive) values of thermal conductivity and Prandtl number are given. The correlation equations of the second method are applicable to a range of H₂-He mixtures consisting of hydrogen volumetric compositions of 1.0-0.75 over a wide range of temperatures of 1000-25,000 K and pressures of 1-31 atm.

The results of both correlating procedures were compared with the detailed results. In addition, the first correlation method is incorporated in the VSL analysis of Moss¹ and comparisons of radiative and convective heating rates obtained with this correlation and a currently used set of transport data are discussed. The required mixture properties computed by the second method are likewise employed in the analysis of Moss¹ and similar heating comparisons are also discussed.

Analysis

Detailed Analysis

The detailed analysis 8 utilizes the most current and reliable data (i.e., the interaction potentials and collision integrals) to compute the transport properties of the individual species and the multicomponent gas mixtures. The significant contribution of Biolsi's analysis for the H₂-He mixture transport properties is a detailed calculation that accounts for all the important two-body collisions (interactions) of the various species in the total mixture. The calculations of the mixture properties are based on rigorous kinetic theory for multicomponent gas mixtures. 9 The contribution of mass diffusion to the thermal conductivity was included which permitted the calculation of the total (frozen plus reactive) thermal conductivity and Prandtl number, similar to the air analysis of Hansen. 10 The binary diffusion coefficient used to

calculate the reactive conductivity was based upon molecular hydrogen diffusing into atomic hydrogen for temperatures less than 7000 K. For temperatures greater than 7000 K, an ambipolar diffusion coefficient ($D_{\rm amb} = 2~D_{\rm H^-H^+}$) was used. For the calculation of the total Prandtl number, the reactive specific heat is based on a reference temperature of 0 K.

Calculated results of the viscosity and total thermal conductivity based on the analysis of Ref. 8 were compared in Figs. 1a and 1b, respectively, of Ref. 11 with the results of Grier ^{12,13} and Yos. ¹⁴ The results were calculated for pure H₂ at one atmosphere pressure for a temperature range of 3000-15,000 K. The results of Biolsi ⁸ and Grier ^{12,13} were shown to be in good agreement, with discrepancies of generally less than 3% noted over the temperature range. The results of Yos ¹⁴ are in similar agreement for temperatures less than 10,000 K. The discrepancies (as large as 100%) with the results of Yos ¹⁴ for temperatures greater than 10,000 K are due in part to the fact that mixture calculations used in Refs. 8, 12, and 13 are based on rigorous kinetic theory for multicomponent gas mixtures while approximate mixing rules were used by Yos. ¹⁴

Correlation Methods

For a radiative flowfield calculation, a detailed transport analysis would substantially increase the amount of computer run time and storage required. Consequently, most flowfield codes employ some approximate technique for computing the individual and/or the mixture transport properties. Such an approximate technique, using correlations based on the results of the detailed analysis, 8 has been developed and is reported herein.

The first correlation (hereafter referred to as method I) is a computational procedure that yields transport properties for the individual species. The correlation is based on an exponential curve fit for each species as a function of temperature and is given as

$$Q_{p,i} = \exp\left(C_i\right) T^{A_i f_n T + B_i} \tag{1}$$

 $Q_{p,i}$ represents the viscosity or thermal conductivity for the individual species and T is the temperature. For the correlation, the temperature range is 1000-25,000 K. The coefficients required in Eq. (1) to compute the desired transport property of the individual species H, H_2 , H $^+$, e $^-$, H_e , and H_e^+ are presented in Table 1. The mixture viscosity and frozen thermal conductivity may be computed by using the semiempirical methods of Wilke 15 and Mason and

Table 1 Coefficients for method I transport correlation
a) Viscosity coefficients

Species	$\mathtt{A_i}$	Вį	c _i
н	4.7622453×10 ⁻²	8.6627216×10 ⁻²	-1.1711139×10 ¹
H_2	7.8201592×10 ⁻²	$-4.4845798 \times 10^{-1}$	-9.1570636×10 ⁰
н+	-8.8065896×10 ⁻¹	1.9417680×10 ¹	-1.1630094×10 ²
e-	-8.8648824×10 ⁻¹	1.9551323×10 ¹	-1.2078068×10 ²
He	4.5837698×10 ⁻²	6.1095003×10 ⁻²	-1.0289207×10 ¹
He ⁺	1.3972250×10 ⁻¹	-7.8183571×10 ⁻¹	-1.5655289×10 ¹

b) Thermal conductivity coefficients

Species	A ₁	Вi	c _i
н	4.7623524×10 ⁻²	8.6638215×10 ⁻²	-9.7106408×10 ⁰
н ₂	7.8165351×10 ⁻²	-4.4788812×10 ⁻¹	-7.8519516×10 ⁰
н+	-8.8063650×10 ⁻¹	1.9417262×10 ¹	-1.1429791×10 ²
e-	-8.8648638×10 ⁻¹	1.9551288×10 ¹	-1.1126648×10 ²
He	4.3851406×10 ⁻²	9.4010190×10 ⁻²	-9.8028246×10 ⁰
He ⁺	1.3949453×10 ⁻¹	-7.7732330×10 ⁻¹	-1.5061806×10 ¹

Saxena, 16 respectively. Note that care should be exercised when using the mixing laws for $\rm H_2$ -He mixtures at temperatures less than 8000 K. A problem may occur due to the ionized species and the electron. The concentrations of these species below 8000 K may be small, but the magnitude of the viscosity of the individual species can be even smaller.

The second correlation (hereafter referred to as method II) is a computational procedure that yields mixture transport properties for equilibrium H_2 -He gas mixtures. The mixture transport properties included in this correlation are viscosity, thermal conductivity, and Prandtl number. For the thermal conductivity and the Prandtl number, correlations are provided for both the frozen and total values. The transport properties were computed employing the detailed analysis for three selected H_2 -He gas compositions (1.0 H_2 -0 He, 0.89 H_2 -0.11 He, 0.75 H_2 -0.25 He) for a temperature range of 1000-25,000 K and pressures of 1-31 atm.

The calculation of a mixture transport property at high temperature may be illustrated as a series of curves, each curve representing a transport property variation with temperature for a given pressure. The development of the method II correlation initially involved collapsing the detailed results for each transport property at each gas composition to the respective data at a pressure of 31 atm as a function of temperature. This collapsing procedure can be generally represented as

$$Q_p^* = C_p Q_p \tag{2}$$

and

$$T^* = C_T T \tag{3}$$

where C_p and C_T are the empirically determined analytic correlation factors for the transport property Q_p and the temperature T, respectively. Thus, Q_p^* and T^* represent the collapsed transport property and temperature, respectively. These collapsed data were curve fitted by an 18th order leastsquares polynomial. The coefficients for the least-squares polynomial applicable to each property for the three selected gas compositions and the correlation factors are presented in Table 2 and Appendix A, respectively. Note that a sensitivity study was not conducted to determine the minimum number of digits required in the coefficients for satisfactory comparison with the detailed results. The procedure for utilizing these coefficients to compute the desired transport properties at the three selected gas compositions is also presented in Appendix A. A more versatile application of method II should provide for the transport property calculation over a range of elemental gas compositions similar to the H2-He thermodynamic correlations of Zoby. 17 This step was accomplished for the range of gas compositions considered herein by collapsing the three least square curve fits representing each gas composition for each property to their respective 100% H₂ curves. The results were collapsed by employing a second set of analytical correlation factors based primarily on the initial volumetric composition of hydrogen. This step of method II can be represented generally as

$$Q' = \bar{C}_p \bar{Q}_p \tag{4}$$

$$T' = \bar{C}_T T^* \tag{5}$$

where \bar{C}_p and \bar{C}_T are the correlation coefficients required to collapse the transport property, \bar{Q}_p , given by the proper 18th order curve fit as a function of temperature to the respective 100% H_2 case. Q'_p and T' represent, respectively, the curve fit value of the transport property and the transformed temperature, T^* , collapsed to the 100% H_2 case. The correlation factors \bar{C}_p and \bar{C}_T are presented in Appendix B. The procedure for utilizing these correlation factors to compute the desired transport property for an arbitrary composition at a given temperature and pressure within the range of the present correlations is also presented in Appendix B. (An

illustration of the steps used in the method II correlation is given in Ref. 11.)

Results and Discussion

The method I correlation as given by Eq. (1) provides a good correlation of the viscosity and thermal conductivity values for the individual species (H, H₂, H⁺, e⁻, He, H_c⁺) as computed by the analysis of Ref. 8. Discrepancies between the correlated and calculated transport values were always less

than 2% over the temperature range where the individual species existed in significant concentration. However, the mixture viscosity and frozen thermal conductivity computed with results of Eq. (1) and existing semiempirical mixing laws ^{15,16} did not generally provide a satisfactory comparison with corresponding detailed results. Differences of 40% were noted in the comparisons at temperatures less than 4000 K and factors of 4 were noted for comparisons at temperatures greater than 7000 K. This poor comparison would indicate a

Table 2 Least squares coefficients for method II transport correlation at three H₂ compositions

b) Frozen mixture conductivity

	100 H ₂	89 H ₂	75 H ₂
a ₁	9.7983696189824x10 ¹	8.8040201860427x10 ¹	7.3162669737662x10 ¹
a ₂	-3.2445029274202x10 ²	-2.9361079946906x10 ²	-2.4539854163259x10 ²
a 3	4.8239171792652x10 ²	4.4170978867050x10 ²	3.7478608633851x10 ²
a ₄	-3.8870282172580x10 ²	-3.5778063369684x10 ²	-3.0604323074419x10 ²
a ₅	1.9490701900986x10 ²	1.8014865744753x10 ²	1.5487762696539x10 ²
a ₆	-6.5182233601302x10 ¹	-6.0415373286122x10 ¹	-5.2105897153804x10 ¹
a ₇	1.5255034991315x10 ¹	1.4169669302275x10 ¹	1.2248019965016x10 ¹
a ₈	-2.5878932072478x10 ⁰	-2.4081853581488x10 ⁰	-2.0852613817543x10 ⁰
a ₉	3.2630412793064x10 ⁻¹	3.0416007414990x10 ⁻¹	2.6377809289674x10 ⁻¹
a 10	-3.1104477962800x10 ⁻²	-2.9040774770462x10 ⁻²	-2.5221016415504×10 ⁻²
a ₁₁	2.2646121015920x10 ⁻³	2.1177166316510x10 ⁻³	1.8416972825058x10 ⁻³
a 12	-1.2645526646291x10 ⁻⁴	-1.1843752403578x10 ⁻⁴	-1.0314054048836x10 ⁻⁴
a ₁₃	5.4049583242917x10 ⁻⁶	5.0700979114816x10 ⁻⁶	4-4213028882378x10 ⁻⁶
a ₁₄	-1.7524596416608x10 ⁻⁷	-1.6464137598477x10 ⁻⁷	-1.4377336621147x10 ⁻⁷
a ₁₅	4.2322907665373x10 ⁻⁹	3.9822636879444x10 ⁻⁹	3.4825068070143x10 ⁻⁹
^a 16	-7.3715919387818x10 ⁻¹¹	-6.9466665379870x10 ^{-1]}	-6.0838873873394x10 ⁻¹¹
a 17	8.7460227273438x10 ⁻¹³	8.2543602882254x10 ⁻¹³	7.2402340390148x10 ⁻¹³
a 18	-6.3242259737378x10 ⁻¹⁵	~5.9777025307740x10 ⁻¹⁵	-5.2515749744832x10 ⁻¹⁵
a ₁₉	2.1024106519038x10 ⁻¹⁷	1.9901880256058x10 ⁻¹⁷	1.7512805392526x10 ⁻¹⁷

d) Total mixture Prandtl number

d) Total mixture Prandtl number			
100 H ₂	89 H ₂	75 H ₂	
a ₁ 6.7647970614119x10 ⁰	6.0809548547083x10 ⁰	5.0377402759899x10 ⁰	
a ₂ -1.8750901973214x10 ¹	-1.6407250278334x10 ¹	-1.2816534638489x10 ¹	
a ₃ 2.3925929894264x10 ¹ a ₄ -1.6871792739266x10 ¹	2.0842689460022x10 ¹ -1.4532490353769x10 ¹	1.6027459304187x10 ¹ -1.0821787338871x10 ¹	
a ₄ -1.6871792739266x10' a ₅ 7.4467157321162x10 ⁰	6.3031562093291×10 ⁰	4.4670725181677x10 ⁰	
a ₆ -2.2182848483167x10 ⁰	-1.8383275883966x10 ⁰	-1,22198635353 48 x10 ⁰	
a ₇ 4.6859756234058x10 ⁻¹	3.7945406451260x10 ⁻¹	2.3342664806267x10 ⁻¹	
a ₈ -7.2585627584456x10 ⁻²	-5.7375126581389x10 ⁻²	-3.2200283540983x10 ⁻²	
a ₉ 8.4347910897364x10 ⁻³ a ₁₀ -7.4647199671363x10 ⁻⁴	6.5052767001459x10 ⁻³ -5.6169260857306x10 ⁻⁴	3.2750452987289x10 ⁻³ -2.4832916838043x10 ⁻⁴	
a ₁₁ 5.0758405806628x10 ⁻⁵	3.7274455749014x10 ⁻⁵	1.4070594437507x10 ⁻⁵	
a ₁₂ -2.6604050806803x10 ⁻⁶ a ₁₃ 1.0720035796996x10 ⁻⁷	-1.9078401575645x10 ⁻⁶ 7.5146047651600x10 ⁻⁸	-5.9136121962055x10 ⁻⁶ 1.8042724781441x10 ⁻⁸	
a ₁₃ 1.0720035796996x10 3 a ₁₄ -3.2896412220712x10 ⁻⁹	-2.2571028937381×10 ⁻⁹	-3.8065776007601x10 ⁻¹⁰	
a ₁₅ 7.5462780849844x10 ⁻¹¹	5.0763519692361x10 ⁻¹¹	4.8995417536405x10 ⁻¹²	
a ₁₆ -1.2526223497432x10 ⁻¹²	-8,2777118072732x10 ⁻¹³	-2.0480301634676x10 ⁻¹⁴	
a ₁₇ 1.4207452076228x10 ⁻¹⁴	9.2436573362240x10 ⁻¹⁵	-4.2132520719453x10 ⁻¹⁶	
a ₁₈ -9.8495088115343x10 ⁻¹⁷	-6,3245732294046x10 ⁻¹⁷	7.1690736186358x10 ⁻¹⁸	
a ₁₉ 3.1477262775551x10 ⁻¹⁹	1.9999104962292x10 ⁻¹⁹	-3.5508369385886x10 ⁻²⁰	

Table 2—Concluded

e) Total mixture conductivity

	100 н ₂	89 H ₂	75 H ₂
a ₁	-1.7730668563976x10 ⁵	-1.7239141397770x10 ⁵	-1.7181367378464x10 ⁵
a ₂	3.4189832372219x10 ⁵	3,3455113326 97 8x10 ⁵	3.3679892735002×10 ⁵
a 3	-2.9145891246444x10 ⁵	-2.8698778889638x10 ⁵	-2.9187035575499x10 ⁵
a ₄	1.4618785216281x10 ⁵	1.4481274461696x10 ⁵	1.4877563586631x10 ⁵
a ₅	-4.8513805530847x10 ⁴	-4.8331206942849x10 ⁴	-5.0150700739467x10 ⁴
a ₆	1.1366949448123x10 ⁴	1.1384651912651x10 ⁴	1.1928321151023x10 ⁴
a ₇	-1.9587206761678x10 ³	-1.9715506718276x10 ³	-2.0851386796279x10 ³
a ₈	2.5502843923388x10 ²	2.5788937071149x10 ²	2.7521084181331x10 ²
a ₉	-2.5543475764290x10 ¹	-2.5941178169019x10 ¹	-2.7922366761324x10 ¹
a 10	1.9902715207567x10 ⁰	2.0293131289924x10 ⁰	2.2022202111715x10 ⁰
a 11	-1.2132150883921x10 ⁻¹	-1.2415691368592x10 ⁻¹	-1.3578387511032x10 ⁻¹
a ₁₂	5.7885210658320x10 ⁻³	5.9439321420548x10 ⁻³	6.5483856733231x10 ⁻³
a ₁₃	-2.1510696166455x10 ⁻⁴	-2.2157361538461x10 ⁻⁴	-2.4580038131573x10 ⁻⁴
a ₁₄	6.1550020435856x10 ⁻⁶	6.3582970520542x10 ⁻⁶	7.0996291654620x10 ⁻⁶
a ₁₅	-1.3288629357021x10 ⁻⁷	-1.3763879082215x10 ⁻⁷	-1.5463179838182x10 ⁻⁷
a ₁₆	2.0924647724746x10 ⁻⁹	2.1725662937548x10 ⁻⁹	2.4549025448250x10 ⁻⁹
a ₁₇	-2.2664344440206x10 ⁻¹¹	-2.3584429618987x10 ⁻¹¹	-2.6794024458555x10 ⁻¹¹
a 18	1.5090000753864x10 ⁻¹³	1.5734704581756x10 ⁻¹³	1.7967135749301x10 ⁻¹³
a ₁₉	-4.6538322329856x10 ⁻¹⁶	-4.8617578245188x10 ⁻¹⁶	-5.5780952546020x10 ⁻¹⁶

need for an adequate mixing law for H2-He gas mixtures.

The method II correlations, as represented by Eqs. (2) and (3), were developed based on results of detailed mixture calculations at 800 pressure-temperature combinations for each property and gas composition. For all the transformed data and the associated least-squares curve fit, the results of an error analysis indicated mean errors less than 0.2% and standard deviations less than 4.5%. Consequently, the method II correlations, based on Eqs. (2) and (3) for the viscosity and both the frozen and total values of the thermal conductivity and Prandtl number, resulted in good comparisons with each of the detailed mixture calculations, 8 with discrepancies of generally less than 10% noted. Equations (4) and (5) provide for a correlation of the transport property over the present range of gas compositions. The results of these correlations were also in good agreement (approximately 15%) with the detailed calculations with maximum errors of approximately 25% noted with these correlations. However, the larger errors usually occurred for comparisons at pressures of 1 atm and temperatures of 18,000 K or greater. A typical comparison of the results of these calculations was presented in Ref. 11.

The form of the method I and method II correlations provides the opportunity to easily employ the results of a detailed transport analysis in flowfield or convective heating formulations. Thus, the method I correlation and the frozen property correlations of method II have been incorporated in the VSL analysis of Moss. 1 Nonradiating, nonblowing convective heating rates were computed using the present correlations and the results were compared in Fig. 5 of Ref. 11. The convective heating rates computed with the currently used set of transport properties employed in Ref. 1 are also presented. This set of transport property data 1 represents a collection of existing transport data for the individual species. Rigorous calculations were not always used to compute the individual species values, and mixture properties were computed by semiempirical mixing laws. The convective heating distributions were computed for a 35 deg hyperboloid at a freestream velocity of 42.88 km/s, a freestream density of 2.54×10^{-4} kg/m³, and a wall temperature of 3954 K.

The convective heating distribution results obtained using the method I transport properties are approximately 60% less than the convective results for the currently used set of transport properties. The comparison demonstrates the significant effect of differences in species transport property data on convective heating. As previously noted, the existing semiempirical mixing laws do not provide satisfactory frozen H_2 -He mixture properties when compared to detailed mixture results. These differences were illustrated in the comparison of the computed convective heating rates by using the frozen mixture property correlations of method II. An increase of approximately 26% in the convective rates based on the transport properties of method I was computed.

The effect of differences in transport properties on radiative heating rates was evaluated by computing radiative nonblowing heating rates for the previously mentioned freestream conditions and body shape. As might be expected, the overall levels of convective rates were reduced in comparison to the nonradiating case and no significant difference was noted in the radiative heating rate results for the different sets of transport data.

An advantage of the method I correlation is that it is not restricted to a limited range of gas compositions. The method also provides accurate definition of the transport properties for the individual species and thus may be applied to nonequilibrium chemistry calculations. Furthermore, the method should not require any more computer time or storage than required by present transport correlations. However, method I requires some technique for computing a desired mixture property. The currently available mixing laws, when used in this method, do not provide satisfactory mixture properties. Method II is based on a detailed equilibrium transport property calculation for H2-He gas mixtures and provides the more reliable equilibrium mixture transport property values over the present range of gas compositions. The method can easily be included in a detailed analysis to calculate the mixture viscosity and the frozen mixture Prandtl number and thermal conductivity. For approximate computational procedures limited to mixture properties, the range of the analysis is usually restricted since correlations of thermodynamic and/or transport properties are typically developed for one gas mixture. 18 However, with method II and a similar thermodynamic correlation, ¹⁷ such restrictions can be alleviated since both correlations provide rapid and versatile techniques of computing the desired properties for a range of H₂-He gas mixtures. Thus, method II is ideally suited for inclusion in the approximate inviscid radiative flowfield code of Zoby4 to provide a convective heating capability for H2-He gas mixtures.

Conclusions

Two distinct correlations of a detailed H₂-He transport property calculation are discussed. The first correlation is applicable to the transport property calculation of individual species. Mixture properties may be computed with semiempirical mixture rules. The second correlation is applicable only to mixture transport property calculations. Mixture transport properties included in this correlation are viscosity, thermal conductivity, and Prandtl number. For the thermal conductivity and Prandtl number, correlations are provided for both the frozen and total values. The second set of correlations are applicable to a range of H₂-He mixtures consisting of hydrogen volumetric compositions of 1.0-0.75 over a range of temperatures of 1000-25,000 K and pressures of 1-31 atm.

The transport property results for the individual species based on the first correlation method were in good agreement with results of detailed calculations. The transport correlation for the individual species is general in nature and may be applied to nonequilibrium calculations. However, results computed for the mixture properties using these species correlations and semiempirical mixing laws were in poor

agreement with corresponding detailed calculations. For the second correlation method, calculated results at the three correlated gas compositions were in very good agreement with results of detailed mixture calculations. Also, the step of the second correlation which provides for the transport property calculation over the range of gas compositions considered herein yielded good comparisons with the detailed results.

Comparisons of convective heating calculations demonstrated the need to base transport property correlations on detailed calculations and the need for reliable mixing laws for H₂-He gas mixtures. A 26% increase in convective heating was noted when the heating calculations were based on the mixture transport property correlations rather than using the individual species transport value and the existing mixing law. Variations of the transport property values had a negligible effect on radiative heating

Appendix A—Correlation Coefficients to Collapse Data

For any high-temperature transport property, the calculated results can be displayed as a family of curves, each curve representing the temperture variation for a given pressure. The initial step of method II correlation was to collapse the data for each transport property at each gas composition to a reference pressure. The reference chosen for this step of the correlation was the transport property computed for a pressure of 31 atm. This procedure of collapsing the data to the 31 atm condition as a function of temperature can be represented by

$$Q_p^* = C_p Q_p \tag{A1}$$

and

$$T^* = (T/1000) C_T \tag{A2}$$

 C_p and C_T are the empirically determined analytic correlation factors for the transport property, Q_p , and temperature, T, respectively. Q_p^* and T^* represent the collapsed transport property and temperature, respectively. The expressions for these factors are given as follows:

Viscosity

Viscosity
$$C_{p} = \frac{10^{5} \left(\frac{T^{*}}{12.5}\right)^{\{0.05[1 + \tanh((T - 13000)/1000)][(1 - \tanh(p - 13)) + (1 - \tanh(p - 5))(1.5 - 5.5(1 - XH2)^{1/2})]\}}{1 + 0.06\ln(p/31)}$$
(A3)

$$C_T = \exp[\ln(31/p)/12.5] \tag{A4}$$

Conductivity (frozen)

Conductivity (frozen)
$$C_{p} = \frac{23.9 \left(\frac{T^{*}}{12.5}\right)^{\{0.5[I + \tanh((T - I3000)/1000)][0.08(I - \tanh(2(p - 3))) + 0.1I5(I - \tanh(p - II))]\}}}{I + 0.06\ell_{h}(p/3I)}$$
(A5)

$$C_T = \exp[\ell_0 (31/p)/12.5]$$
 (A6)

Prandtl number (frozen)

$$C_p = I + 0.014 \ln (31/p) \left[I + \tanh \left(\frac{(T - 10000)}{500} \right) \right] + 0.05 \left[I + \tanh \left(\frac{(T - 15000)}{500} \right) \right] (I - \tanh (p - 5)) (I - 4(I - XH2)^{1/2})$$
(A7)

$$C_T = \exp\left[\ln\left(31/p\right)/12.5\right] \tag{A8}$$

Prandtl number (total)

$$C_p = I + 0.0875 \left(I + \tanh\left(\frac{(T - 15000)}{500}\right) \right) (I + \tanh(p - 7)) (I - 2.5(I - XH2)^{1/2})$$
(A9)

$$C_T = \exp[\ln(31/p)/12.5]$$
 (A10)

Conductivity (total)

$$C_{p} = [I + AA(AB + AC + AD + AE + AF + AG + AH + AI + AJ + AK + AL + AM + AN + AO + AP)]23.9$$
(A11)

where

$$AA = 0.5(I + \tanh((T - 2250)/25)), \quad AB = 0.5(\exp[\ln(p/3I)/6.5] - I)(I + \tanh((2750 - T)/25))$$

$$AC = 0.00625\tanh(p - I)(I + \tanh(19 - p))(I + \tanh((3250 - T)/25))(I - \tanh((2750 - T)/25))$$

$$AD = 0.125(\exp[\ln(p/3I)/14.5] - I)(I - \tanh((3250 - T)/25))(I + \tanh((3750 - T)/25))(I + \tanh(9 - p))$$

$$AE = -0.01625(I - \tanh(p - I7))(I + \tanh((4250 - T)/25))(I - \tanh((3750 - T)/25))$$

```
AF = 0.25(1 - \tanh((4250 - T)/25))(1 + \tanh((4750 - T)/25))[0.35(1 + \tanh(2(1-p)))]
   +0.055 \tanh (3-p) \tanh (p-1) (1+\tanh (19-p))
AG = 0.25(1 - \tanh((4750 - T)/25))(1 + \tanh((5250 - T)/25))[0.5(1 - \tanh(p - 3))]
   +0.5(\exp\{\ln(31/p)/16\}-1)(1+\tanh(2(p-4)))]
AH = 0.25(1 - \tanh((5250 - T)/25))(1 + \tanh((5750 - T)/25))[0.2(1 + \tanh(13 - p)) + (\exp[\ln(31/p)/14] - 1)]
AI = 0.25(1 - \tanh(5750 - T)/50))(1 + \tanh((6250 - T)/50))[0.1(1 - \tanh(p - 21)) + \exp\{\ln(31/p)/15\} - I]
AJ = 0.25(1 - \tanh((6250 - T)/50))(1 + \tanh((6750 - T)/50))[0.085(1 - \tanh(p - 25))]
   +0.105 \tanh (p-1) (1 + \tanh (10-p))
AK = 0.0125(1 - \tanh((6750 - T)/50))(1 + \tanh((7250 - T)/50))(1 - \tanh(p - 25))
AL = 0.0075(1 - \tanh((7250 - T)/50))(1 + \tanh((7750 - T)/50))(1 + \tanh(0.5(9 - p)))\ln(31/p)
AM = 0.00265 \ln (31/p) (1 - \tanh ((10750 - T)/50)) (1 + \tanh ((12250 - T)/50)) (1 + \tanh (17 - p))
AN = 0.00375 \ln(p/31) \left(1 - \tanh((14000 - T)/100)\right) \left(1 + \tanh((16500 - T)/1000)\right) \left(1 + \tanh(4(14 - p))\right)
AO = 0.5(1 + \tanh((T - 18000)/1000))[0.02\ln(31/p) + 0.5(1 + \tanh(7-p))(\exp{\ln(31/p)/12.5} - 1)]
AP = 0.025(1 - \tanh((19000 - T)/500))(1 + \tanh(2(21 - p)))
C_T = 1 + 0.25(1 + \tanh((8000 - T)/250))(1 + \tanh((T - 2750)/25))(\exp{\{\ln(31/p)/15\}} - 1)
   +0.25 \left(\exp\left\{\ln\left(31/p\right)/12.5\right\}-1\right) \left(1-\tanh\left((8000-T)/250\right)\right) \left(1+\tanh\left((T-11000)/250\right)\right)
                                                                                                                              (A12)
```

The coefficients for the least-squares polynomial and the correlation factors for the total thermal conductivity are applicable over the temperature range of 2500-25,000 K. The correlation for this transport property from 1000-2500 K is given as (note this result was not presented in Ref. 11):

$$k_t = 418.4(4.133 \times 10^{-10}T^2 - 2.533 \times 10^{-7}T + 8.5 \times 10^{-4})/(1 + A1 \cdot A2)$$
 (A13)

where

$$A1 = 0.5(1 + \tanh((T - 23500)/75)), \quad A2 = (\exp\{\ln(p/31)/6.5\} - 1)$$

Calculation of Transport Property, Q_p

The transport data collapsed by the proper correlation constants [Eqs. (A3-A12)] are fit by an 18th order least-squares polynomial as a function of the temperature, T^* . For freestream hydrogen volumetric compositions of 1.0, 0.89, or 0.75, the procedure for utilizing these curve fits to compute the desired transport property at some temperature and pressure is given as follows:

- 1) Given some T(k) and p (atm) and freestream hydrogen volumetric composition of 1.0, 0.89, or 0.75.
- 2) Compute C_T and T^* .
- 3) Use applicable polynomial coefficients and value of T^* to compute \bar{Q}_p . \bar{Q}_p is the collapsed transport property given by the least-squares polynomial.
 - 4) Compute C_n .
 - 5) Transport property, Q_p , is computed by dividing \bar{Q}_p by C_p .

Appendix B—Correlation Coefficients for Arbitrary Gas Compositions

A more versatile correlation of the transport properties should allow for a variable gas composition. Such a technique has been developed over the range of hydrogen volumetric compositions of 1.0-0.75. This correlation was accomplished by collapsing the three least squares curve fits representing each gas composition for each property to the respective $100\%~H_2$ curve. This correlation can be represented as

$$Q_p' = \bar{C}_p \bar{Q}_p \tag{B1}$$

$$T' = \bar{C}_T T^*, \quad \bar{T} = 1000 T^*$$
 (B2)

 \bar{C}_p and \bar{C}_T are the correlation factors for an arbitrary gas composition. \bar{Q}_p is the transport property given by the 18th order curve fit and T^* is the transformed temperature computed by Eq. (A2). Q'_p and T' represent the transport property and temperature, respectively, collapsed to the 100% H₂ case. The expressions for these factors are given as follows:

Viscosity

$$\bar{C}_p = (1 + 1.06 \ln(XH2)) \left(\frac{T'}{12.5}\right)^{-0.8(1 - XH2)^{\frac{1}{2}}} \left[(1 + \tanh((T' - 15200)/125)) + (1 + \tanh((T'' - 17100)/125)) + 1.28(1 + \tanh((T'' - 19500)/125)) \right]$$
(B3)

$$\bar{C}_T = \exp\{-0.333(1 - XH2)\}$$
 (B4)

T'' = 1000T'

Conductivity (frozen)

$$\tilde{C}_p = I + 0.2 \ln (XH2) \left[-2 + \left((I + \tanh ((6500 - \tilde{T})/250) \tanh ((\tilde{T} - I2500)/500))/2 \right) \right. \\
+ 2.5 (I + \tanh ((\tilde{T} - I7500)/250)) + (I + \tanh ((\tilde{T} - I2500)/250) \tanh ((I5500 - \tilde{T})/250)) \right]$$
(B5)

$$\tilde{C}_T = I$$
 (B6)

Prandtl number (frozen)

$$\bar{C}_p = (1 + B1 + B2) \left(\exp\{0.25(1 - XH2)^{1/2}\} \left(1 + \tanh((\tilde{T} - 17000) / 500) \right) \right)$$
 (B7)

where

$$BI = 0.2(1 - XH2)^{1/2} (1 + \tanh((\bar{T} - 15000)/500))$$

and

$$B2 = 0.5(1 + \tanh((\bar{T} - 23500)/250)) \exp\{6(1 - XH2)\} \left[0.45(1 - XH2)^{\frac{1}{2}} + (XH2 - 1)\left(\frac{T'}{12.5}\right)^{(1 - XH2)}\right]$$

$$\bar{C}_T = I - 0.5(1 - \exp\{-0.55(1 - XH2)^{\frac{1}{2}}\}) (I + \tanh((T - 15000)/250))$$
 (B8)

Prandtl number (total)

$$\bar{C}_p = 1 - B3(B4 + B5) - B6(B7) - B8(B9) + B10(B11)$$
 (B9)

where

$$B3 = 0.5(1 - \exp\{-(1 - XH2)\}), \quad B4 = 1 + \tanh((3000 - \bar{T})/500), \quad B5 = 1 - \tanh((6500 - \bar{T})/500)$$

$$B6 = 0.5 \left[1 - \left(\frac{\tilde{T}}{13000} \right)^{-2(1 - XH2) \frac{1}{2}} e^{(1 - XH2)} \right], \quad B7 = 1 - \tanh\left(\left(13500 - \tilde{T} \right) / 250 \right)$$

$$B8 = 0.5 (1 - XH2)^{1/2} \left[\left(\frac{T'}{12.5} \right)^{XH2} - 2.05 (1 - XH2)^{1/2} \right], \quad B9 = 1 - \tanh \left((18000 - \bar{T})/250 \right)$$

$$B10 = ((1 - XH2)^{2}/3) \left(\frac{T'}{12.5}\right)^{(1 - XH2)/2}, \quad B11 = 0.5(1 - \tanh((21500 - \bar{T})/250)), \quad \bar{C}_{T} = \exp\{-(1 - XH2)\}$$
 (B10)

Conductivity (total)

$$\tilde{C}_{p} = [I - BA - BB - BC - BD - BE - BF - BG - BH] \tag{B11}$$

where

$$BA = 0.25(1 - \exp\{-(1 - XH2)/3\})(1 + \tanh((\bar{T} - 3250)/125))(1 + \tanh((4250 - \bar{T})/125))$$

$$BB = 0.25(1 - \exp\{(1 - XH2)/3\})(1 + \tanh((\bar{T} - 5750)/125))(1 + \tanh((6250 - \bar{T})/125))$$

$$BC = 0.25(1 - \exp\{(1 - XH2)/2\})(1 + \tanh((\bar{T} - 6250)/125))(1 + \tanh((7250 - \bar{T})/125))$$

$$BD = 0.25(1 - \exp\{(1 - XH2)/3\})(1 + \tanh((\tilde{T} - 7250)/125))(1 + \tanh((7750 - \tilde{T})/125))$$

$$BE = 0.25(1 - \exp\{(1 - XH2)/3\})(1 + \tanh((\bar{T} - 9750)/125))(1 + \tanh((13750 - \bar{T})/125))$$

$$BF = 0.25(1 - \exp\{(1 - XH2)/2\})(1 + \tanh((\bar{T} - 13750)/125))(1 + \tanh((15750 - \bar{T})/125))$$

$$BG = 0.25 (1 - \exp\{(1 - XH2)/1.5\}) (1 + \tanh((\bar{T} - 15750)/125)) (1 + \tanh((23250 - \bar{T})/125))$$

$$BH = 0.5(1 - \exp\{(1 - XH2)/2\})(1 + \tanh((23250 - \bar{T})/125))$$

$$\vec{C}_T = 1 \tag{B12}$$

In this appendix, correlation coefficients [Eqs. (B3-B12)] which collapse the curve fits of the transformed transport data at three selected gas compositions to the pure H_2 case have been presented. The procedure for computing a desired transport property at a given value of T, p, and XH2 within the range of the present correlations is given as follows. (Note that when T^* , T', C_p , and \bar{C}_p are computed, the desired value of XH2 is employed.)

Viscosity:

- 1) Use polynomial coefficients for viscosity at 100% H_2 condition and value of T' to compute Q'_p . T' is computed using Eqs. (A2), (A4), (B2), and (B4).
 - 2) Compute C_p , Eq. (A3), and \bar{C}_p , Eq. (B3).
- 3) Viscosity $(N \cdot s/m^2)$ is computed by dividing Q'_p by product of C_p and \bar{C}_p .

Conductivity (frozen)

- 1) Use polynomial coefficients for frozen conductivity at 100% H_2 condition and value of T^* to compute Q'_p . T^* is computed using Eqs. (A2) and (A6).
 - 2) Compute C_p , Eq. (A5), and \tilde{C}_p , Eq. (B5).
- 3) Frozen conductivity (W/m-K) is computed by dividing Q'_p by product of C_p and \bar{C}_p .

Prandtl number (frozen):

- 1) Use polynomial coefficients for frozen Prandtl number at 100% H_2 condition and value of T' to compute Q'_p . T' is computed using Eqs. (A2), (A8), (B2), and (B8).
 - 2) Compute C_p , Eq. (A7), and \bar{C}_p , Eq. (B7).
- 3) Frozen Prandtl number is computed by product of Q_p' and \bar{C}_p divided by C_p .

Prandtl number (total):

- 1) Use polynomial coefficients for total Prandtl Number at 100% H_2 condition and value of T^* to compute Q_p' . T^* is computed using Eqs. (A2) and (A10).
 - 2) Compute C_p , Eq. (A9), and \bar{C}_p , Eq. (B9).
- 3) Total Prandtl number is computed by dividing Q'_p by product of C_p and \bar{C}_p .

Conductivity (total):

- 1) Use polynomial coefficients for total conductivity at 100% H₂ condition and value of T^* to compute Q'_p . T^* is computed using Eqs. (A2) and (A12).
 - 2) Compute C_p , Eq. (A11), and \bar{C}_p , Eq. (B11).
- 3) Total conductivity (W/m-K) is computed by dividing Q'_p by product of C_p and \bar{C}_p .

4) Note that for the total thermal conductivity from 1000-2500 K Eq. (A13) can be used over the range of gas composition considered herein.

References

¹Moss, J. N., "Stagnation and Downstream Viscous Shock Layer Solutions with Radiation and Coupled Ablation Injection," AIAA Paper 74-73, Jan. 1974.

²Kumar, A., Tiwari, S. N., and Graves, R. A. Jr., "Effects of Small Angle of Attack on the Radiating Viscous Shock Layer Solutions for Jovian Entry," AIAA Paper 78-908, May 1978.

³Sutton, K., "Characteristics of Coupled Nongray Radiating Gas Flows with Ablation Product Effects About Blunt Bodies During Planetary Entries," Ph.D. Thesis, North Carolina State University, 1973.

⁴Zoby, E. V., Sutton, K., Olstad, W. B., and Moss, J. N., "An Approximate Inviscid Radiating Flow Field Analysis for Outer Planet Entry Probes," AIAA Paper 78-189, Jan. 1978.

⁵Moss, J. N., Anderson, E. C., and Bolz, C. W., Jr., "Aerothermal Environment for Jovian Probes," AIAA Paper 76-469, July 1976.

⁶Sutton, K., "Radiative Heating About Outer Planet Entry Probes," AIAA Paper 75-183, Jan. 1975.

⁷ Anderson, E. C., Moss, J. N., and Sutton, K., "Turbulent Viscous Shock Layer Solutions with Strong Vorticity Interaction," AIAA Paper 76-120, Jan. 1976.

⁸Biolsi, L., "Transport Properties in the Jovian Atmosphere," *Journal of Geophysical Research*, Vol. 83, No. A3, March 1978, pp. 1125-1131.

⁹Hirschfelder, J. D., Curtiss, C. F., and Bird, R. B., *Molecular Theory of Gases and Liquids*, John Wiley, New York, 1954.

¹⁰ Hansen, C. F., "Approximations for the Thermodynamic and Transport Properties of High-Temperature Air," NASA TR R-50, 1959.

¹¹Zoby, E. V., Graves, R. A., Jr., Moss, J. N., Simmonds, A., and Kumar, A., "Correlations for Determining Transport Properties of Hydrogen-Helium Gas Mixtures at Temperatures From 1000 to 25,000 K," AIAA Paper 79-0034, Jan. 1979.

¹²Grier, N. T., "Calculation of Transport Properties and Heat Transfer Parameters of Dissociating Hydrogen," NASA TN D-1406, Oct. 1962.

¹³ Grier, N. T., "Calculation of Transport Properties of Ionizing Atomic Hydrogen," NASA TN D-3186, April 1966.

¹⁴Yos, J. M., "Transport Properties of Nitrogen, Hydrogen, Oxygen, and Air to 30,000°K," AVCO Corp., Rept. No. RAD-TM-63-7, March 1963.

¹⁵ Wilke, C. R., "A Viscosity Equation for Gas Mixtures," *Journal of Chemical Physics*, Vol. 18, No. 4, April 1950, pp. 517-519.

¹⁶ Mason, E. A. and Saxena, S. C., "Approximate Formula for the Thermal Conductivity of Gas Mixtures," *The Physics of Fluids*, Vol. 1, No. 5, 1958.

¹⁷Zoby, E. V., Gnoffo, P. A., and Graves, R. A. Jr., "Correlations for Determining Thermodynamic Properties of Hydrogen-Helium Gas Mixtures at Temperatures From 7000 to 35,000 K," NASA TN D-8262, Aug. 1976.

¹⁸ Falanga, R. A. and Olstad, W. B., "An Approximate Inviscid Radiating Flow Field Analysis for Sphere-Cone Venusian Entry Vehicles," AIAA Paper 74-758, July 1974.